

# Tavis-Cummings model and collective multi-qubit entanglement in trapped ions

A. Retzker,<sup>1,2,\*</sup> E. Solano<sup>†,3,4</sup> and B. Reznik<sup>5</sup>

<sup>1</sup>*Institute for Mathematical Sciences, Imperial College London, SW7 2PE, UK*

<sup>2</sup>*QOLS, The Blackett Laboratory, Imperial College London, Prince Consort Rd., SW7 2BW, UK*

<sup>3</sup>*Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Strasse 1, D-85748 Garching, Germany*

<sup>4</sup>*Sección Física, Departamento de Ciencias, Pontificia Universidad Católica del Perú, Apartado Postal 1761, Lima, Peru*

<sup>5</sup>*Department of Physics and Astronomy, Tel-Aviv University, Tel Aviv 69978, Israel*

We present a method of generating collective multi-qubit entanglement via global addressing of an ion chain performing *blue* and *red* Tavis-Cummings interactions, where several qubits are coupled to a collective motional mode. We show that a wide family of Dicke states and iradiant states can be generated by single global laser pulses, unitarily or helped with suitable postselection techniques.

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## I. INTRODUCTION

Multi-partite entangled states play an important role in quantum information. They are useful in various quantum information applications, such as in Heisenberg-limited spectroscopy [1], secure communication [2], and various schemes related to "one-way" quantum computing [3]. Fresh theoretical developments on the generation of multipartite entangled states show that sequential techniques may prove to be general and practical for building arbitrary multi-qubit states [4]. For instance, a recent experiment [5] has realized a *W* state of eight qubits, encoded in the internal ionic levels, by performing a sequence of two-qubit gates on different ion pairs. However, given a set of available interactions in a physical system, there are particular families of entangled states that could be built globally and in fewer steps [6]. In the context of cavity QED (CQED), for example, the coupling of a single cavity mode with a two-level atom, the Jaynes-Cummings model (JC), can be extended to the *N* atom case, leading to the Tavis-Cummings model, with different dynamics and entanglement features [7, 8].

In this article, we study methods of generating specific classes of multi-qubit entangled states in trapped ions with collective interactions, which are potentially faster and more efficient than individual techniques. They consist of two key ingredients: firstly, the use of global rather than individual addressing of ions and, secondly, the presence of *invariant subspaces*, i.e., combined (vibronic) internal and motional finite subspaces that are *closed* with respect to certain dynamical operations.

In Sec. II, we describe realistic collective vibronic in-

teractions coupling the internal degrees of freedom of *N* ions with a collective motional mode. Specifically, we consider the blue and red excitation versions of the Tavis-Cummings model, taking distance from usual predictions in the Dicke model. In Sec. III, we study the invariant subspaces, associated with the proposed interactions, in the search of classes of multipartite entangled states that may be efficiently generated. It will turn out that one of them is the family of symmetric Dicke states [9, 10, 11], from which the *W* state is just a one-excitation particular case. In Sec. IV, we consider the family of entangled states that could be generated by means of purely unitary global operations and, in Sec. V, the ones that could be generated by using postselection.

## II. COLLECTIVE MAPS

Let us consider *N* ions in a linear Paul trap, cooled down to their collective motional ground state. We will not concentrate on a specific experimental setup [12], and our derivations could be applied to any ion-trap device. The free-energy Hamiltonian,  $H_0$ , describing the *N* two-level ions and their motion around their equilibrium positions is

$$H_0 = \frac{\hbar\omega_0}{2} \sum_{n=1}^N \sigma_z^n + \hbar \sum_{j=1}^N \nu_j a_j^\dagger a_j. \quad (1)$$

Here,  $\sigma_z^n$  are *z*-components of Pauli spin vectors describing the two-levels with energy gap  $\omega_0$ , while  $a_j$  and  $a_j^\dagger$  are the annihilation and creation operators for the normal modes with frequency  $\nu_j$ . The interaction between the internal degrees of freedom of each ion and a collective motional mode can be induced by laser light of frequency  $\omega$ , yielding [13]

$$H_{\text{int}}^n = \hbar\lambda_n \sigma_x^n \cos(kx_n - \omega t + \phi_n). \quad (2)$$

<sup>†</sup>Present address: Physics Department, Ludwig-Maximilian University, Munich, Germany, enrique.solano@physik.lmu.de

\*Electronic address: a.retzker@imperial.ac.uk

Here,  $\lambda_n$  is the coupling strength between the laser and the  $n$ -th ion,  $\sigma_x^n$  are  $x$ -components of Pauli vectors,  $k$  is the laser wave vector,  $x_n$  is the displacement operator with respect to the equilibrium position, and  $\phi_n$  is the phase of the laser at the location of the  $n$ -th ion.

We will study the case of homogeneous laser excitation,  $\lambda_n = \lambda$ ,  $\forall n$ , and of near resonant coupling,  $\omega \approx \omega_0$ . For the sake of simplicity, we will also consider all  $\phi_n = 0$ , although this may play an important role when making experimental considerations. In this case, the Hamiltonian in the interaction picture, after a rotating-wave-approximation (RWA) with respect to the two internal levels, reads [13, 14]

$$H^I = \frac{\hbar\lambda}{2} \sum_n (\sigma_+^n e^{-i\delta t} \exp(ik \sum_j b_{nj} \sqrt{\frac{\hbar}{2m\nu_j}} (a_j^\dagger e^{i\nu_j t} + a_j e^{-i\nu_j t})) + \text{H.c.}), \quad (3)$$

where  $b_{nj}$  denote the amplitudes of the  $j$ -th normal mode of the ion chain in the position expansions,  $\delta = \omega - \omega_0$ , and  $m$  is the ion mass. In the Lamb-Dicke limit, where all Lamb-Dicke parameters  $\eta_j = k\sqrt{\frac{\hbar}{2m\nu_j}}$  are small, the exponential can be expanded and set for a RWA with respect to the phonon field. In that case, when the laser frequency is tuned to a particular collective motional sideband frequency,  $\omega = \omega_0 \pm \nu_j$ , we obtain blue and red sideband transition Hamiltonians

$$H_{\text{blue}}^j = \hbar\tilde{\lambda}_j \sum_n b_{nj} (\sigma_+^n a_j^\dagger + \sigma_-^n a_j), \quad (4)$$

$$H_{\text{red}}^j = \hbar\tilde{\lambda}_j \sum_n b_{nj} (\sigma_+^n a_j + \sigma_-^n a_j^\dagger), \quad (5)$$

where  $\tilde{\lambda}_j = \eta_j \lambda / 2$ . The interaction of Eq. (5) appears naturally in the context of CQED, where a bunch of atoms interact inhomogeneously with a cavity mode and the counter-rotating terms are neglected in the RWA. The dynamics in Eq. (4) is not usual in CQED but can be easily engineered in trapped ions. Only when  $\nu_j$  corresponds to the center-of-mass (COM) mode frequency  $\nu_1$ , we have  $b_{n1} = b_1$ , and we can define the collective angular momentum operators  $L_+ = \sum_n \sigma_+^n$  and  $L_- = \sum_n \sigma_-^n$ . In that case, we could rewrite the Hamiltonians of Eqs. (4) and (5) as

$$H_{\text{blue}}^1 = \hbar\tilde{\lambda}_1 (L_+ a^\dagger + L_- a) \quad (6)$$

$$H_{\text{red}}^1 = \hbar\tilde{\lambda}_1 (L_+ a + L_- a^\dagger), \quad (7)$$

where  $\tilde{\lambda}_1 = b_1 \tilde{\lambda}$ . The dynamics associated with the Hamiltonian of Eq. (7) is named after Tavis and Cummings [7], who developed the first analytical solutions for this model. When we consider a motional mode different from the center-of-mass one, we could always define  $L_+^j =$

$\sum_n b_{nj} \sigma_+^n$  and  $L_-^j = \sum_n b_{nj} \sigma_-^n$ , but these operators do not satisfy the usual angular momentum algebra. If we define  $L_z \equiv \sum_n \sigma_z^n$  and  $L^2(j) \equiv L_z^2 + \frac{1}{2}(L_+^j L_-^j + L_-^j L_+^j)$ , with  $j = 0, 1, \dots, N-1$ , we get

$$\begin{aligned} [L_z, L_\pm^j] &= \pm L_\pm^j, \\ [L_z, L^2(j)] &= 0, \\ [L_\pm^j, L^2(k)] &\neq 0, \quad \{j, k\} \neq 0. \end{aligned} \quad (8)$$

In fact,  $L_\pm^j$  can still be used to lower and raise the quantum numbers of  $L_z$ , but they do not commute with  $L^2(j)$ . For the case of the center-of-mass mode, where all commutations relations are satisfied, we shall denote the eigenstates of  $L^2(1)$  and  $L_z$  by  $|l, m\rangle$ , with  $l = N/2, N/2 - 1, \dots, l > 0$ , and  $-l \leq m \leq l$ . States  $|l, m\rangle$  are known as the Dicke states [9, 10, 11].

### III. INVARIANT SUBSPACES

Hamiltonian  $H_{\text{red}}^j$  conserves the total number of spin and phonon excitations, and commutes with the excitation number operator  $\hat{R} \equiv \sum_m a_m^\dagger a_m + L_z + N/2$ , while Hamiltonian  $H_{\text{blue}}^j$  conserves the difference between the spin and phonon excitations, hence, it commutes with  $\hat{B} \equiv \sum a_m^\dagger a_m - L_z + N/2$ . It is therefore possible to consider vibronic subspaces with a fixed number of excitations associated with  $\hat{R}$  or  $\hat{B}$ . If we concentrate on the case  $H = H_{\text{red}}^j$ , we have the eigenstates  $|r, \alpha\rangle$  of  $\hat{R}$ , where  $r = 0, 1, 2, \dots$ , and  $\alpha$  denotes other degeneracy lifting quantum numbers. We then obtain the block diagonal structure  $H_{\text{red}}^j = \oplus_{r=0}^\infty H_{\text{red}}^j(r)$ . The dynamical evolution that is generated by  $H_{\text{red}}^j$  leaves the subspaces invariant.

We proceed to discuss certain examples of such invariant subspaces, for example, the one associated with the case  $j = 1$ . The smallest eigenvalue of  $\hat{R}$ ,  $r = 0$ , corresponds to the state  $\mathcal{H}_{r=0} = \{|l = N/2, m = -N/2\rangle|0\rangle\}$ , i.e., all atoms in their ground state and no phonons in the system. For the case  $r = 1$ , we have

$$\mathcal{H}_{r=1} = \mathcal{H}_{l=N/2} \oplus \mathcal{H}_{l=N/2-1}, \quad (9)$$

where

$$\mathcal{H}_{l=N/2} = \{|N/2, -N/2\rangle|1\rangle, |N/2, -N/2+1\rangle|0\rangle\} \quad (10)$$

and

$$\mathcal{H}_{l=N/2-1} = \{|N/2-1, -N/2+1, \alpha=1\rangle|0\rangle, \dots, |N/2-1, -N/2+1, \alpha=N-1\rangle|0\rangle\}. \quad (11)$$

The quantum number  $\alpha = 1, \dots, N-1$ , lifts the  $(N-1)$ -fold degeneracy of the states with  $l = -N/2 + 1$ . Hence, values of  $\alpha$  enumerate the different angular momentum

multiplets. It is important to stress that  $H_{\text{red}}^1$  does not mix the different multiplets and, since  $L_-|N/2 - 1, -N/2 + 1, \alpha\rangle = 0$ , there are no further transitions. This does not follow merely from the conservation of  $\hat{R}$ , which does not forbid transition between the state  $|N/2 - 1, -N/2 + 1, \alpha\rangle|0\rangle$ , which has terms with one excited atom, and a state with one excited phonon. This non-mixing property of the multiplets reflects the effect of quantum *irradiance* [9, 15, 16]. The construction of higher  $r$ -number subspaces is straightforward. For instance, for  $r = 2$  we shall have  $\mathcal{H}_{r=2} = \mathcal{H}_{l=N/2} \oplus \mathcal{H}_{l=N/2-1} \oplus \mathcal{H}_{l=N/2-2}$ , etc.

A key point in the present work is the use of subspaces which are bidimensional. In this simple case, the evolution of the system resembles that of the well known Rabi oscillations. For example, let us consider the  $r = 1$  invariant subspace  $\mathcal{H}_{r=1, l=N/2}$ . We can start with the non-entangled state containing one phonon and with all the internal spins in their ground states. When we turn on the Hamiltonian  $H_{\text{red}}^1$  we obtain an oscillation between the states

$$|N/2, -N/2\rangle|1\rangle \leftrightarrow |N/2, -N/2 + 1\rangle|0\rangle. \quad (12)$$

State  $|N/2, -N/2 + 1\rangle$  is a symmetric combination of  $N$  terms,  $(|\uparrow\downarrow\downarrow\cdots\rangle + |\downarrow\uparrow\downarrow\cdots\rangle + \cdots + |\cdots\downarrow\downarrow\uparrow\rangle)/\sqrt{N}$ , known as the  $W$  state. Similarly, we could make use of the invariant space  $\mathcal{H}_{r=1, l=N/2-1, \alpha}$  and, in that case, we would have the following oscillation

$$|N/2, -N/2 + 1\rangle|1\rangle \leftrightarrow |N/2, -N/2 + 2\rangle|0\rangle. \quad (13)$$

In the general case, the invariant subspaces can be of higher dimension, for instance if we start with  $n$  phonons in the multiplet  $l = N/2$ , the relevant states for  $r = n$  becomes, up to rotations induced by  $H_{\text{red}}$ ,

$$|-N/2\rangle|l\rangle \leftrightarrow |-N/2 + 1\rangle|l - 1\rangle \leftrightarrow \cdots \leftrightarrow |-N/2 + l\rangle|0\rangle. \quad (14)$$

So far, we have discussed invariant subspaces which are connected with the Dicke states and the collective angular momentum operators with  $j = 1$ . By tuning the laser to couple other motional collective modes, we can access other  $j$  subspaces. As we discuss in the next section, it is sometimes helpful to combine several steps, and in each step to couple a different phonon normal mode. For instance, we can start with the state that contains two different phonon excitations

$$|\downarrow\downarrow\cdots\downarrow\rangle|1\rangle_i|1\rangle_j, \quad (15)$$

couple first the internal levels with the phonon in mode  $i$  and later with the phonon in mode  $j$ . This process

connects us with the state  $L_+^j L_+^i |l = N/2, m = -N/2\rangle$ . It is useful to see that in this type of transitions we have

$$\begin{aligned} L_-^j L_+^i |N/2, -N/2\rangle &= L_-^j \sum_n b_{ni} |\downarrow\downarrow\cdots\downarrow \underbrace{\uparrow}_n \downarrow\cdots\downarrow\rangle \\ &= \left( \sum_n b_{ni} b_{nj} \right) |\downarrow\downarrow\cdots\downarrow\rangle = n \delta_{ij} |\downarrow\downarrow\cdots\downarrow\rangle, \end{aligned} \quad (16)$$

where in the last step we used the orthogonality of the normal modes.

#### IV. DETERMINISTIC CREATION OF ENTANGLED STATES

With the use of  $H_{\text{red}}$ , many relevant states can be created. We start with the state

$$|N/2, -N/2\rangle|1\rangle_{\nu_1} = |\downarrow\downarrow\downarrow\cdots\downarrow\rangle|1\rangle_{\nu_1}, \quad (17)$$

where the  $j$ -th mode is occupied by a single phonon, and the internal state is not entangled. A  $W$ -state  $|W_1^N\rangle \equiv |N/2, -N/2 + 1\rangle$  can be created by applying a single collective  $\pi/2$ -pulse on the state of Eq. (17). This can be easily understood by recalling that the above initial state belongs to the bidimensional Hilbert space,  $\mathcal{H}_{r=1, l=N/2} = \{|N/2, -N/2\rangle|1\rangle, |N/2, -N/2 + 1\rangle|0\rangle\}$ . In principle, by precise control of the duration and intensity of the laser pulse, a  $W$  state can be created between a large number of ions. In fact, a  $W$ -state shared by eight ions has been created recently using a multi-step sequential procedure based on individual ionic addressing [5]. In the present proposal, we would require the previous preparation of a single phonon in the COM mode and the application of a single homogeneous global laser pulse. A related scheme in the context of quantum dots was discussed recently by Taylor *et al.* [17].

It is also possible to generate deterministically higher-excitation Dicke states using other bidimensional invariant subspaces. The  $r = 2$  subspace  $\mathcal{H}_{r=2, l=N/2-1, \alpha}$  is a two-dimensional space that is spanned by the states  $|N/2 - 1, -N/2 + 1\rangle|1\rangle \sim W_1^N$  and  $|N/2 - 1, -N/2 + 2\rangle|0\rangle \sim W_2^N$ . The first state above is equivalent, up to local operations, to the  $W_1^N = |N/2, -N/2 + 1\rangle$ , while the second state, contains terms with two excited atoms and is equivalent, up to local transformations, to the second Dicke state  $|N/2, -N/2 + 2\rangle = W_2^N$ . The construction of  $W_2^N$  can therefore proceed as follows. We first obtain as described above  $W_1^N$ , using a single pulse. In the second step, we transform  $W_1^N \rightarrow |N/2 - 1, -N/2 + 1\rangle$  by changing locally the phases of each ion. This step requires local addressing implementing local rotations. In the final step, we add a single phonon and apply again  $H_{\text{red}}^1$  to obtain  $W_2^N$ , the second member of the subspace

$r = 2$ . Unfortunately, it seems that for higher-excitation Dicke states, e.g.  $W_3^N$ , this “climbing the ladder” method requires also some interaction between the qubits. To overcome this difficulty we shall discuss other methods.

We consider next extended examples of coupling to other modes and show that they can be used for generating *irradiant states* [9, 15, 16]. We start with the state  $\psi_0 = |N/2, -N/2\rangle|1\rangle_{\nu_j}$  involving one phonon in the  $j$ -th mode and all the internal levels in their ground state, then, we apply the Hamiltonian  $H_{\text{red}}^j$ . The conservation of  $\hat{R}$  restricts the possible evolution to the subspace of states with  $r = 1$ , i.e., to the states  $L_+^j \psi_0$  and  $L_-^k L_+^j \psi_0$ , with  $k = 1, 2, \dots, N$ . However, we notice from Eq. (16) that only terms with  $k = j$  do not vanish, hence, the evolution leads to Rabi oscillations in the bidimensional Hilbert space  $\{\psi_0, L_+^j \psi_0\}$ . In this way, we can generate the family of entangled irradiant states of the form

$$L_+^j |\downarrow \downarrow \dots \downarrow\rangle, \quad j = 1, 2, \dots, N. \quad (18)$$

Irradiant states are states that do not emit photons and are thus more robust to decoherence than radiant states. In our case, this property is due to the relation in Eq. (16). Since the coupling to the electromagnetic field is through the  $L_{\pm}$  operators, as it is for the phonon field, the resultant states are irradiant [9]. For the the case of two spins, the resulting state is the EPR state. The experimental feasibility of irradiance and superradiance in ion traps was discussed and demonstrated by De Voe and Brewer [15].

Having produced certain irradiant states, we can use them as a starting point for the deterministic generation of an additional class of states. Irradiant states introduce other bidimensional invariant subspaces. Since  $L_- |\psi_{\text{irr}}\rangle = 0$ , the subspace  $\{|\psi_{\text{irr}}\rangle|1\rangle, L_+ |\psi_{\text{irr}}\rangle|0\rangle\}$  is an invariant subspace of Hamiltonian  $H_{\text{red}}$  and, therefore, the second state can be produced by Rabi flipping. This is a new kind of entangled state which is a superposition of states with two spins in the upper state,

$$\begin{aligned} \Psi \propto & \left( \underbrace{|\uparrow \downarrow \dots \downarrow \uparrow \downarrow \dots \downarrow\rangle}_{\text{odd}} + \text{perm} \right) \\ & - \left( \underbrace{|\downarrow \uparrow \dots \downarrow \uparrow \downarrow \dots \downarrow\rangle}_{\text{even}} + \text{perm} \right). \end{aligned} \quad (19)$$

For the case of four spins the outcome of this process is a GHZ state. First, we apply  $H_{\text{red}}^3$ , which couples the internal states with the higher collective mode,  $j = N - 1 = 3$ , and create the irradiant state

$$L_+^3 |\downarrow \downarrow \dots \downarrow\rangle = |\uparrow \downarrow \downarrow \downarrow\rangle - |\downarrow \uparrow \downarrow \downarrow\rangle + |\downarrow \downarrow \uparrow \downarrow\rangle - |\downarrow \downarrow \downarrow \uparrow\rangle. \quad (20)$$

In the next stage, we apply the  $j = 0$  red Hamiltonian

and get

$$L_+^0 L_+^3 |\downarrow \downarrow \dots \downarrow\rangle = |\uparrow \downarrow \uparrow \downarrow\rangle - |\downarrow \uparrow \downarrow \uparrow\rangle, \quad (21)$$

which is, up to a local operation, a GHZ state.

## V. CREATION OF ENTANGLED STATES WITH POSTSELECTION

In the previous section, we have discussed deterministic schemes for producing irradiant states as well as the lowest Dicke states (including the  $W$  state). However, the full family of Dicke states could not be generated using only collective unitary transformations. In the present section, we present another approach which is useful for producing the full set of Dicke states

$$\begin{aligned} W_k^N &= \binom{N}{k}^{-1/2} \left( \underbrace{|\uparrow \uparrow \dots \uparrow \downarrow \downarrow \dots \downarrow\rangle}_k + \text{perm} \right) \\ &\equiv |N/2, -N/2 + k\rangle. \end{aligned} \quad (22)$$

The properties of the Dicke states may be of considerable interest in quantum information, and have been discussed recently by different authors [10], [18], [11], [19], [2]. It can be shown that the von Neumann entanglement entropy, with respect to a bi-partite split of  $N$  qubits in a Dicke state, increases with  $k$  and saturates gradually for large  $k$  values. The behavior of the (mixed state) entanglement between two qubits [19] can be evaluated by considering, for example, the negativity which increases, almost linearly with  $k$ .

The basic idea behind our approach is that while a collective unitary transformation cannot be used to create any Dicke state, a suitable choice of the initial phonon state can bring us very close to our goal. In this scheme, however, there will be always a small error due to mixing with other states. Therefore, unlike the previous examples, we propose to postselect the phonon state in order to be certain that the desired Dicke state was produced.

In order to create the Dicke state  $W_k^N$ , we begin by preparing the initial state  $|N/2, -N/2\rangle|k\rangle_{\nu_0}$ . We then apply the time evolution of the Hamiltonian  $H_{\text{red}}^j$ , which takes this state into the  $r = k$  invariant subspace. It turns out that at a certain time the probability distribution will be sharply peaked around a state with zero phonons and  $W_k^N$  for the internal levels. By measuring the number of the phonons it is then possible to remove the admixture of  $W_k^N$  with other states. A procedure to create and measure the number operator in an ion trap was introduced by different authors [20, 21, 22]. Experimentally, motional Fock states were already produced in the lab [23], although those techniques required a series of consecutive Rabi flips.

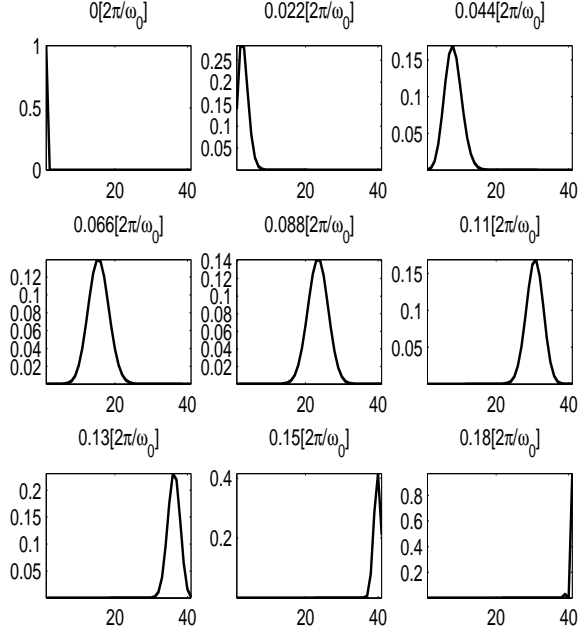


FIG. 1: Population of various states in Hilbert space as a function of time for 100 spins and 40 phonons. It can be seen that the population of the last state is maximal.

The crucial ingredient in our proposed mechanism is that the purity of the state, prior to postselection, is high. The fact that the state containing zero phonons in Eq. (10) is produced with high probability is shown numerically below. The intuition behind this is that there is an analogy between these subspaces and the angular momentum subspaces of  $L^2$ , though the commutation relations are different. The  $L_x$  operator rotates the spin about the  $x$  axis, producing states with  $L_z = \pm l$  with a probability one. In order to model this dynamics an analogy could be made between this dynamics in Hilbert space and the dynamics of a particle travelling between sites with different coupling strength. Since the couplings are higher at the middle and lower at the edges, the probabilities are maximal at the edges, see Figs. 1 and 2. We therefore expect the Hamiltonian  $H_{\text{red}}$  to rotate the state between the first and last state in Eq. (14) with a probability close to unity.

We make now some further considerations concerning our scheme based on postselection. The only states which are created with high probability are  $|l_z = m\rangle$ , where  $m$  is the number of phonons. This is due to the fact that the last state is created with high probability, therefore, the number of phonons in the first state determines the final state. The population of states in the subspace of

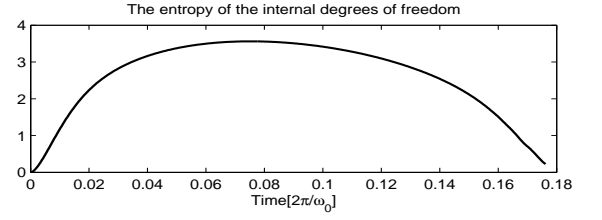


FIG. 2: The entropy of the internal degrees of freedom as a function of time for 100 spins and 40 phonons. It can be seen that the final state is nearly pure (see Fig. 1).

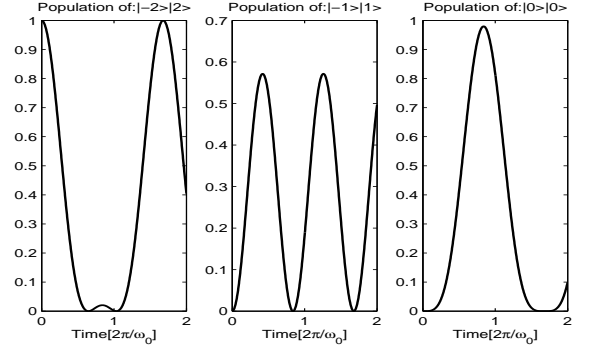


FIG. 3: The probability of states  $|−2\rangle|2\rangle$ ,  $|−1\rangle|1\rangle$ ,  $|0\rangle|0\rangle$  as a function of time.

Eq. (14) starting with  $|−2\rangle|2\rangle$  are shown in Fig. 3, where we observe that for specific times the desired state is obtained with high probability. A similar thing is observed in Fig. 4 starting with  $|−5\rangle|5\rangle$ . In spite of the fact that the number of excitations is not negligible compared to the number of spins, the purity of the final state is considerably high. This observation may prove very useful for generating  $W_k^N$  states.

In order to increase the purity of the final state the number of phonons has to be measured and the vacuum state postselected. To achieve that goal, we consider a recently proposed technique [22] for sorting a desired motional Fock state  $|N\rangle$  out of any motional distribution. This technique is based on a suitably designed vibronic scheme in a single ion, allowing for a restricted dynamics inside a chosen selected JC subspace  $\{|g\rangle|N+1\rangle, |e\rangle|N\rangle\}$ . To adapt it to our present work, we would need an additional idle ion inside the chain, coupled to the motional mode of interest and specifically assigned to postselection purposes. Together with the additional necessity of individual ion addressing for the sake of manipulation and measurement, these requirements for the idle ion are at reach by the state-of-the-art present technology in trapped ions [5]. The proposed scheme described hitherto can also be applied to create motional number states

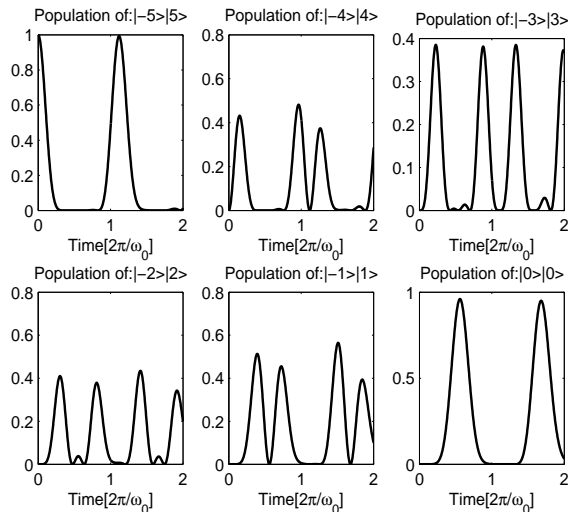


FIG. 4: The population of the various terms. It can be seen that except for the first state, only the last state approaches a value close to 1. The first state is a state with 10 spins down and 5 phonons

via the Hamiltonian  $H_{\text{blue}}$ , which will rotate the state in the proper subspace. Postselecting the spin state will yield the Fock state state  $|N\rangle$  and the number of spins measured up would indicate the number of motional excitations  $N$ .

## VI. CONCLUSIONS

In conclusion, we have presented methods of producing entangled states using homogenous global laser coupling

in trapped ion systems. We have considered two schemes, one based on purely (deterministic) unitary operations and the other one based on an ulterior (probabilistic) postselection. Both schemes use the fact that the Tavis-Cummings model, in its blue- and red-excitation versions, possesses invariant subspaces. In the deterministic case, the global laser pulses produce the desired entangled states after rotations in the associated bidimensional invariant subspaces. In the probabilistic case, the allowed rotations produce edge states that are very close to the desired entangled states, requiring a highly efficient post-selection technique. We believe that all proposed schemes are realistic and at within reach using present state-of-the-art technology in trapped ions.

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